# Buckingham-PI theorem

Dimensional analysis

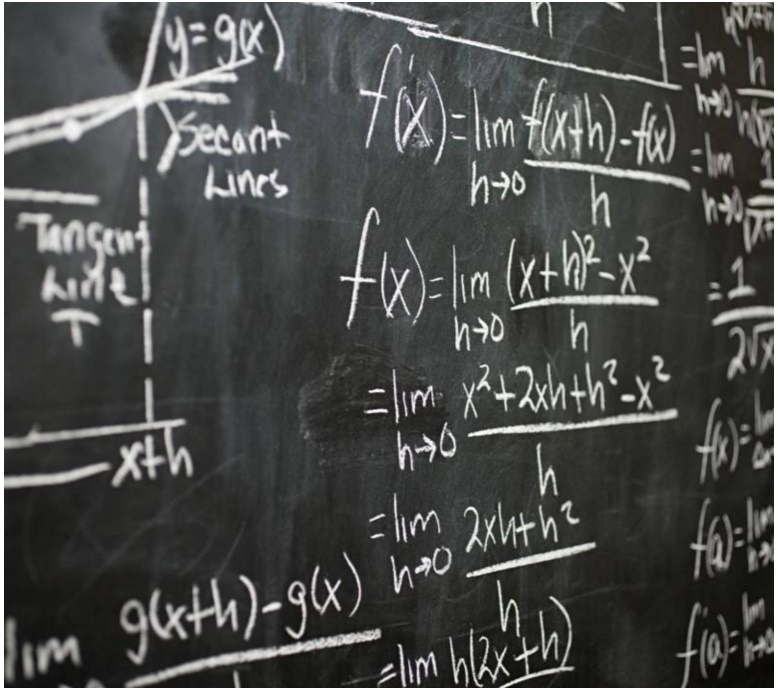
Fracture of materials

# Statement

"If there is a physically meaningful equation involving a certain number ***n*** **physical variables**, then the original equation can be rewritten in terms of a set of $p = n - k$ **dimensionless parameters** $\pi_1, \pi_2, \ldots, \pi_p$ constructed from the original variables, **where** ***k*** **is the number of physical dimensions involved."**

# Why is it useful ?

- Find laws and equations based on physical variables
- Get a better intuition of the important parameters of a system
- Form precise laws from experiments (or theories)
- NB: here π is not 3.1415 …. BUT a dimensionless parameter !



# Steps of the theorem

1. Determine the N important system parameters (physical variables) for what we want to find
2. Build the matrix of the physical variables/dimensions and find its rank K
3. Determine the number of dimensionless groups p = N – K
   - N is the number of physical variables (speed, density, pressure...)
   - K is the number of physical dimensions (kg, m, s...) -> Always take base units for conveniance !
4. Build the p dimensionless groups
5. Find the equation for the physical parameter you are interested in !
   - If there is only one group, only one experiment will be necessary to find a numerical value of the dimensionless parameter π
   - If there is multiple group, multiple experiments will be necessary to find the link between them.

# Exemple





- We aim to find a relationship for the time t it takes for an object initially at rest to fall a certain distance h, under the influence of gravity g. (We neglect friction)

- Step 1 :
  - What are the physical variables ?
  - What are their physical dimensions ?

  - $t = [s]$
  - $h = [m]$
  - $g = [a] = [m/s^2]$

  - $N = 3$

# Example

- Step 2 : build the matrix with :
  - Physical variables in columns
  - Physical dimensions in row

| | t | g | h |
|---|---|---|---|
| s | 1 | -2 | 0 |
| m | 0 | 1 | 1 |

- This is a matrix of rank K = 2 !
- Step 3 : Determine the number of dimensionless parameters p
  - P = N – K = 3 – 2 = 1

EPFL

# Example

- Step 4 : Chose and build the p dimensionless parameters
  - We need only 1 : since we want to find t at the end, let's take it as a basis ! (we could also search h for a given t …)
  - We need to balance t with the other N-P physical variables to make π dimensionless

$$\Pi = t g^{-a} h^{-b} \quad (\Pi \text{ is dimensionless})$$

  - For easier resolution, let's replace the physical variables with their dimensions

$$1 = T^{1} \left(L/T^{2}\right)^{-a} L^{-b} \quad \text{with } T = \text{time and } L = \text{distance}$$

  - We can solve this independently for each dimension !
    - second, T : 0 = 1 + 2a
    - meter, L : 0 = -a - b

    - a = -1/2
    - b = 1/2

$$\Longrightarrow \quad \pi = t * \sqrt{\frac{g}{h}}$$

# Example

- Step 5 : let's find the relation for t !
  - If you know t, g and h, how many experiments will you need to do to determine the numerical value of π ?

$$\pi = t * \sqrt{\frac{g}{h}}$$

  - After one experiment, we find $\pi^2 = 2$ !
  - By isolating t, we find :

$$t = \pi\sqrt{\frac{h}{g}} = \sqrt{\frac{2h}{g}}$$

  - Which is the same relation we would find by resolving kinetics with acceleration = g and initial speed = 0 !

- **What if there is more than one dimensionless parameter ? (p > 1)**

- You need to make more experiments, varying the value of the physical variables to find this relation :

$$\pi_1 = f(\pi_2, \pi_3, \dots)$$

- In our case, if there was more than one dimensionless parameter (for example, adding air drag/resistance), we would need to solve by experiments :

$$t\sqrt{\frac{g}{h}} = f(\pi_2, \pi_3, \dots)$$

- Be careful the relation f could be non-trivial ! (For example, **the Reynold's number is a dimensionless parameter**, influencing other equations through this f in complex ways !)

# Any question ?



- Exercices 1, 2 and 4 will be about Buckingham-Pi

- Exercice 3 will be about wave propagation